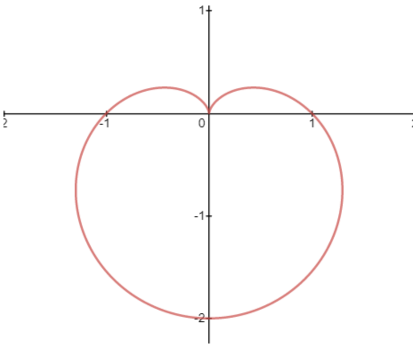
Advanced Mathematics Support Programme®

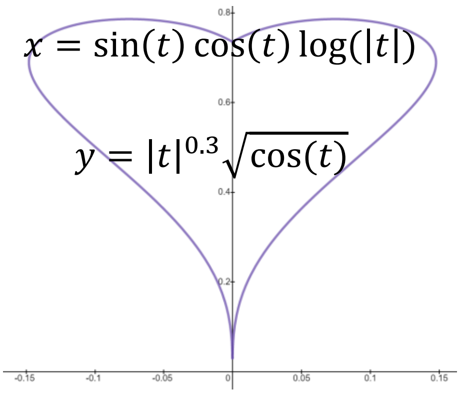
Rearranging Factorising

Did you know?

- Being able to express equations in different forms gives us different information
- Later we'll be looking at information needed to sketch graphs
- If you continue your maths studies to A Level Further Maths, you will draw graphs such as these

$r = 1 - sin\theta$

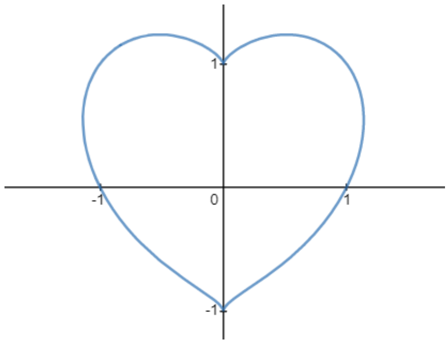




$x = \sin(t)\cos(t)\log(|t|)$

$y = |t|^{0.3}\sqrt{\cos(t)}$

$(x^2 + y^2 - 1)^3 - x^2y^3 = 0$



Futher Factorising 1

1. The equation of a line is given as
   i. $3y + 4x - 2 = 0$.
   b. What is the gradient of the line?

2. A rectangle has area A, length y and width $x - 2$. Write an expression for the length of the rectangle, y, in terms of A and x

3. Make x the subject of:
   a. $ax - y = z + bx$

4. The equation of a line is given as
   i. $5(\mathrm{b} - \mathrm{p}) = 2(\mathrm{b} + 3)$

5. John says the first step to rearranging
   a. $\frac{x-a}{f} = 3g$ is to add $a\ to\ 3g$. Is he right? Explain your answer.

6. Make a the subject of
   a. $5(a - t) = 3(a + x)$

7. Make x the subject of
   a. $ay + x = 4x + xb$

8. Make x the subject of
   a. $2\pi\sqrt{x + t} = 4$

Further Factorising 2

1. Make y the subject of
   $xy + 6 = 7 - ky$

2. Find an expression for the area of a rectangle with length, $(y - x)$ and width, $(x - 2)$

3. Rewrite your expression in Q2 to have y expressed in terms of A and x

4. Make y the subject of
   $\frac{4}{y} + 1 = 2x$

5. Displacement can be expressed as
   i. $s = ut + \frac{1}{2}at^2$

   Express a in terms of s, u and t

6. Make y the subject of $\sqrt{by^2 - x} = D$

7. The area of a trapezium has formula
   i. $A = \frac{1}{2}\left(\frac{a+b}{h}\right)$

   Express h in terms of A, a and b

8. Make t the subject $b(t + a) = x(t + b)$

Equivalent quadratics

Sort the expressions below in to 4 sets of 4 equivalent expressions

$x^2 - 25$	$2x^2 - 2$
$(x+5)(x+6) - x - 55$	$(x+5)(x-5)$
$2(x^2 - 1)$	$(x+5)^2 - 10x - 50$
$2(x+3)(x-1)$	$2(x+1)(x-1)$
$(x+5)^2 - 50$	$2(x+2)^2 - 4x - 14$
$2x^2 + 4x - 6$	$(x+5)(x-5) + 10x$
$2(x+1)^2 - 8$	$(x-5)(x+6) - x + 5$
$x^2 + 10x - 25$	$2(x+1)^2 - 4(x+1)$

Mean squares

- Take two positive values greater than 1
- Find the mean of the two values
- Square it

THEN

- Take the same two values
- Square them
- Find the mean of the squares

Which value is greater?

Is this always true?

Can you prove it?

Hint

- Try out several examples
- Is one expression always bigger than the other?
- Next try using x and y instead.
- If you subtract one expression from the other, can you work out if it's positive or negative?

Difference of numeric squares

Problem 1

Mrs Gryce was asked to calculate 18×12 by Mr Lo who had forgotten his calculator and was doing some marking.

Mrs Gryce quickly responded

"Well, that's just $15^2 - 9$ which is 216"

Mr Lo was amazed.

- **How did she know so quickly what the answer was?**

Problem 2

Use the fact that 3 x 4 = 12

Can you quickly work out a value for $(3.5)^2$?

- **Can you see a connection between the previous question and this one?**

The Quadratic Formula

We've all used the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- But where does it come from?
- Can you prove why the quadratic formula works?

Rearrange these steps in order to prove the quadratic formula

$$ax^2 + bx + c = 0 \longrightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\left(x + \frac{b}{2a}\right) = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$	$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	$ax^2 + bx = -c$
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	$\left(x + \frac{b}{2a}\right) = \pm\sqrt{\left(\frac{b^2 - 4ac}{4a^2}\right)}$	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

Match the steps below with the algebra above for a slightly easier version

Step 1: Subtract c from both sides

Step 2: Divide both sides by a

Step 3: Complete the square on the left hand side

Step 4: Add $\frac{b^2}{4a^2}$ to both sides

Step 5: Make the right hand side into a single expression

Step 6: Take the square root of both sides

Step 7: Simplify the denominator on the right hand side

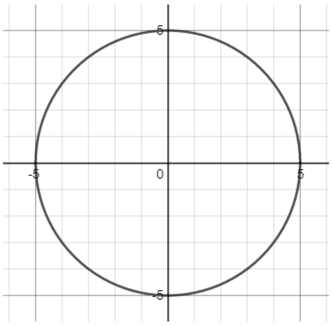
Step 8: Subtract $\frac{b}{2a}$ from both sides

Step 9: You now have the quadratic formula!

Equations of Circles

$$x^2 + y^2 = 25$$

Represents a circle with centre (0,0) and radius 5



Generally, the equation of a circle with centre (0,0) and radius r can be written as

$$\boldsymbol{x^2 + y^2 = r^2}$$

- **What happens if the centre is not (0,0)?**

Let's have a look at this equation: $x^2 + 4x + y^2 - 6y = 12$

We can rearrange this by completing the square separately for the x terms and y terms

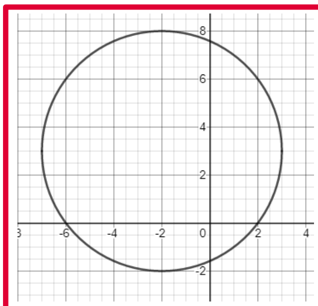
$$x^2 + 4x = (x+2)^2 - 4 \text{ and } y^2 - 6y = (y-3)^2 - 9$$

So $$x^2 + 4x + y^2 - 6y = 12$$

Can be written as $$(x+2)^2 - 4 + (y-3)^2 - 9 = 12$$

$$(x+2)^2 + (y-3)^2 - 13 = 12$$

$$(x+2)^2 + (y-3)^2 = 25$$



$$(x+2)^2 + (y-3)^2 = 25$$

Represents a circle with Centre (-2,3) and radius 5

- Can you find the centre and radii of these circles by rearranging into the form

$$(x+a)^2 + (y-b)^2 = r^2$$

$x^2 - 8x + y^2 - 2y = 19$	$x^2 + 6x + y^2 - 10y = 15$